Effect of particle shape and angularity on dilation of granular soils: a discrete element approach

Effet de la forme et angles de particules sur la dilatation de sols granulaires: approche d'un élément discret

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ABSTRACT

The Discrete Element Method (DEM) was first introduced by Cundall and Strack (1979) to model granular soil particles as discs in 2D and spheres in 3D. Ashmawy et al (2003) developed the Overlapping Rigid Clusters (ORC) method to capture particles actual shape. This paper presents the results of a study performed to explore the effect of particle shape and angularity on strength and dilatancy properties of granular soils using DEM utilizing the ORC method. Triaxial tests under pure shear conditions were modeled for different particle shape and angularity groups under different initial conditions. The developed stress-strain relationships were used to explore the dependency of maximum volumetric strain, maximum dilation angle, and the resulted residual shear strength angle on shape and angularity factors was explored. The numerical study concluded that the critical state lines, for different shape and angularity groups, were shifted upward (more dilation) compared to circular particles. The critical state lines showed significant dependency on interparticle frictions. It was concluded that there is no one-to-one equivalency between interparticle friction and shape or angularity. Instead, the interparticle friction must be continuously altered as function of confining pressure and void ratio to achieve the required effect.

RÉSUMÉ

La méthode d'élément discret était tout d'abord présentée par Cundall et Strack (1979) au modèle de sols granulaires sous forme de disques en 2D et sphère en 3D. Docteur Ashmawy et Al (2003) a développé la méthode de faisceaux rigides de recouvrement (ORC) pour capturer la forme actuelle de ces particules granulaires. Les tests Triaxial, sous les conditions de cisaillement, étaient modelés. Les particules simulées étaient divisées sous différentes formes de particules et de groupes d'angles. Différentes formes et groupes d'angles étaient testés sous différentes conditions. Les relations de cisaillement soumis à une contrainte de cisaillement et volumétrique étaient développées pour les différents groupes circulaires à particules. L'étude numérique a conclut que des lignes critiques pour différentes formes et groupes d'angles, décalé vers le haut, (davantage de dilatation), comparé aux particules circulaires. Les lignes critiques montrent une dépendance significative sur les frictions entre particules. Il a été conclut qu'il n'y a pas de d'équivalence linéaire entre les frictions entre particules et formes d'angles. Au lieu de cela, la friction entre particules doit être changée sans interruption de pression étroite, et éviter le rapport pour obtenir l'effet désiré.

Keywords : Modeling, Discrete Element, Dilatancy, Angularity, Shape Factor, Triaxial, Pure Shear

1 INTRODUCTION

Professor Osborne Reynolds (1885) first defined dilatancy as a unique fundamental property for the granular media, which does not exist in any known fluid or solid. Dilatancy is the change in volume corresponds to shearing granular materials. Reynolds (1885) showed that dense sands expand whereas loose sands contract during shear to failure. The magnitude of dilation depends on soil density and confining pressure (Houlsby 1991). Rowe (1962) introduced his theory of dilatancy. Research studies had proofed that the relation is not unique, instead, it is density and confining pressure dependent.

The discrete element method (DEM) has been used to explore the fundamental granular soil properties such that strength and dilatancy. Ting et al. (1995) presented the results of a DEM study on the influence of particle shape on strength and deformation of two-dimensional ellipse-based particles. Ni et al. (2000) studied the effect of micro-properties of granular materials on shear strength and dilation characteristics through 3-D discrete numerical simulations of the direct shear test.

Shape, the first-order morphologic property, is used to characterize the gross form of a particle and it is mostly defined in terms of three perpendicular axes. Angularity, the second-order property, expresses the number and sharpness of corners on the particle surface. Surface roughness, the third-order property, reflects the number, size and sharpness of the asperities along the particle surface and on the corners. Different techniques have been utilized to define shape, angularity, and surface roughness.

2 MODIFIED PARTICLE SHAPE FACTOR

Sukumaran (1996) and Sukumaran and Ashmawy (2001) used the particle's two-dimensional projection to characterize its shape and angularity. The outline of a two-dimensional projection of a particle can be quantified numerically by discretizing the perimeter, which approximates the true particle shape by an equivalent polygon. The sampling angle is a critical factor in calculating the shape and angularity factors. Small sampling angles capture particle angularity, whereas large sampling angle should be utilized if the overall shape is the main concern. In order to be able to identify the effect of particle shape and angularity separately on the dilatancy of granular soils, the shape and angularity factors should be totally independent. Increasing the sampling angle has a significant effect on the shape factor. A sampling angle of 90˚ was proposed by the authors with a 9˚ rotation angle. In addition to separating the effect of shape and angularity, the 90˚ intervals can be envisioned as two perpendicular axes, which represent a true global shape measurement. The resulted modified shape and angularity factors were clearly independent.

Eight soils that include 99 particles were used in the numerical model. The modified shape factors for all particles varied between 29.22 and 93.21 with an average of 48.32, whereas the angularity factors varied between 3.59 and 61.94 with an average of 20.73. Particles were divided into groups according to their shape and angularity factors as shown in Table 1. For each group, a subroutine was written to the software PFC^{2D} using the program language "fish" in order to create the corresponding clumps. The clumps were created according to the ORC method (Ashmawy et al. 2003).

Table 1. Shape and angularity factors for different groups

Particles Group	SF range, (Percent)	SF average, (Percent)	AF range, (Percent)	AF average, (Percent)
AF1	30 -73	40.00	05 - 10	7.50
AF2	32 - 74	54.00	20 - 25	22.50
AF3	37 - 60	50.00	40 - 45	42.50
SF1	30 - 35	32.50	05 -27	13.00
SF2	50 - 55	52.50	05 -44	25.00
SF3	70 - 75	72.50	07 -27	20.00
SF1(AF 15-25)	30 - 35	32.50	15 - 25	20.00
SF2(AF 15-25)	50 - 55	52.50	15 - 25	20.00
SF3(AF 15-25)	70 - 75	72.50	15 - 25	20.00

3 NUMERICAL SIMULATIONS

Among typical loading conditions for evaluating soil strength, the triaxial test is the most common laboratory method. Dilatancy can be defined, with respect to triaxial or biaxial testing conditions, as the ratio of plastic volumetric strain increment to the plastic deviator strain increment. However, it is known that there is a volumetric change associated with the increase of confining pressure during the triaxial test. To this end, the volumetric strain measured during triaxial testing of granular soils is attributed not only to changes in shear stresses, but also to variations in mean total stress. In order to obtain volumetric strain that is solely due to shear, either simple shear or pure shear testing conditions should be applied. Pure shear may be applied in triaxial configuration by simultaneously varying the vertical and horizontal stresses in equal magnitudes and opposite directions, which technique was used throughout the current simulations to study the effect of particle shape and angularity on dilatancy.

3.1 *Sample preparation and consolidation*

The particles were generated by radius expansion and the number of the particles was set to satisfy sample dimensions, particle size range, and target porosity. The walls used to confine and load and after initial compaction, the lateral walls were given stiffnesses substantially lower than particles stiffness to provide soft boundary and ensure uniform stresses. The angular substitution scheme was then invoked to regenerate the angular particles according to the desired particle shape and angularity groups. After the clumps were created, strains were determined by tracking the position of the walls. Stresses on each wall were computed by dividing the total force acting on the wall by the sample length/width. Stresses acting on the sample in each direction were obtained by taking the average of the stresses acting on each set of opposing walls. The sample was isotropically consolidated to the desired confining pressure by moving the walls inward with a controlled velocity using a servo "fish" function. The consolidation procedure usually involves volumetric change that may be expansion or compression, depending on the desired confining pressure and porosity. The volume change corresponds to applying the confining pressure on circular particles was determined and saved. In order to ensure a unique porosity after the consolidation stage for all soils, samples that involve a volumetric change different from that of the discs were brought back to the same porosity by performing cyclic consolidation. After consolidation, the current conditions were stored as initial conditions in order to begin loading stage.

3.2 *Pure shear loading*

In the numerical simulations, pure shear loading was applied by releasing the top and bottom platens from the servo control function and apply constant inward velocity. In order to ensure a stable solution, the velocity was increased gradually until the target velocity is achieved. The top and bottom platens then continued to move at a constant velocity to the end of the test. Throughout the solution, the increase in the vertical stress was measured and target horizontal stress was set to decrease by the same amount, which ensures pure shear loading condition. The loading terminated at axial strain around 30% to ensure that critical state have been reached. In most tests, the critical state was reached before 30% axial strain but the loading continued to ensure that instability didn't occurred at very high strains. The confining pressure, shear stress, axial strain, and volumetric strain were tracked and saved for further analysis.

3.3 *Simulation program*

The pure shear test was performed on groups AF1, AF2, AF3, SF1, SF2, SF3, SF1(AF 15-25), SF2(AF 15-25), and SF3(AF 15-25) under the same conditions. Different sample states were considered. The state parameter introduced by Been and Jefferies (1986) is an established parameter to define the sample position with respect to the critical state line. The state parameter determines how far the current condition is from the critical state. The numerical simulations were performed on the same shape and angularity groups with three different initial state parameters. Confining pressures of 1.0MPa, 500KPa, and 250KPa with corresponding porosities of 0.14, 0.2, and 0.26, respectively, were used. The interparticle friction coefficient was set to 0.5, which corresponds to an angle of interparticle friction of 25°. The performed numerical simulations are summarized in Table 2. The effect of interparticle friction angle was also studied by performing the test on a an assembly of circular particle using friction coefficients of 0.25 – 2.50, which correspond to interparticle friction angles of 14° - 68°.

Table 2. Simulation program

Group	$P^{/}$ = 1.0 MPa & n = 0.14	$P^{/}$ = 500 KPa & n = 0.20	$P^{/}$ = 250 KPa & n = 0.26
AF1	√	√	√
AF2	√	√	√
AF3	√	√	√
SF1	√	N/A	N/A
SF2	√	N/A	N/A
SF3	√	N/A	N/A
SF1 (AF 15-25)	√	√	√
SF2 (AF 15-25)	√	√	√
SF2 (AF 15-25)	√	√	√
Discs (f = 0.25)	√	√	√
Discs (f = 0.50)	√	√	√
Discs (f = 0.75)	√	√	√
Discs (f = 1.00)	√	√	√
Discs (f = 1.25)	√	√	√
Discs (f = 1.50)	√	√	√
Discs (f = 1.75)	√	√	√
Discs (f = 2.00)	√	√	√
Discs (f = 2.25)	√	√	√
Discs (f = 2.50)	√	√	√

4 NUMERICAL RESULTS

For every confining pressure and the corresponding voids ratio and number of particles, the shear strain versus shear stress and volumetric strain were plotted for every particle shape and angularity group and compared to those of circular particles. Figures 1 and 2 are examples of the stress-strain plots.

The results defined a peak shear strength followed by a residual value. The volumetric strain plots showed dilation to occur throughout the test with higher rate of dilation being at low shear strains. Beyond the peak strength, the rate of dilation begins to decrease until the critical/steady state is reached. Both modified shape and angularity factors were plotted against the maximum volumetric strain and dilation as well as residual shearing resistance angles and trend line were fitted to the correlations with R^2-values mostly above 95%.

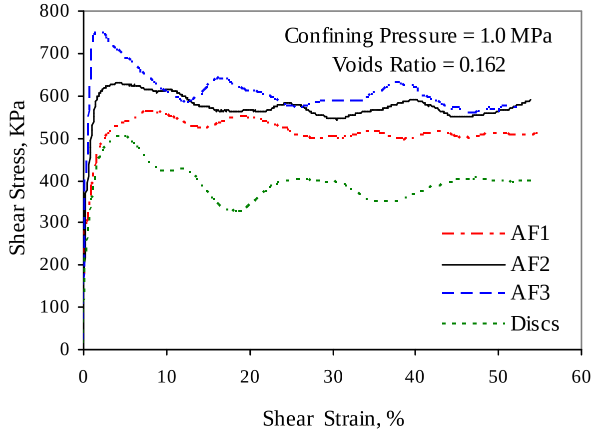

Figure 1. Shear stress-shear strain curves for different angularity groups

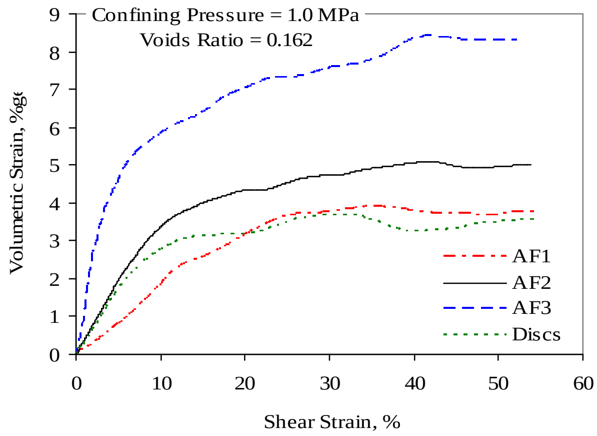

Figure 2. Volumetric vs shear strain for different angularity groups

4.1 *Critical state line and particle shape characteristics*

The results confirmed that strength and dilatancy of granular soils depend on particle shape and angularity. The observed changes in dilatancy characteristics for different groups are not attributed solely to particle angularity or shape. Confining pressure and the corresponding initial void ratio contributed to the behavior as well. To isolate the effect of particle shape and angularity, the critical state lines for different groups were identified in the ν-$p^{/}$ space. The specific volume (ν) was plotted against the confining pressure ($p^{/}$) for different angularity and shape groups, and the results (Figures 3 and 4) showed that the critical state lines for different angularity and shape groups were shifted above that of circular particles with more pronounced effect for the angularity factor.

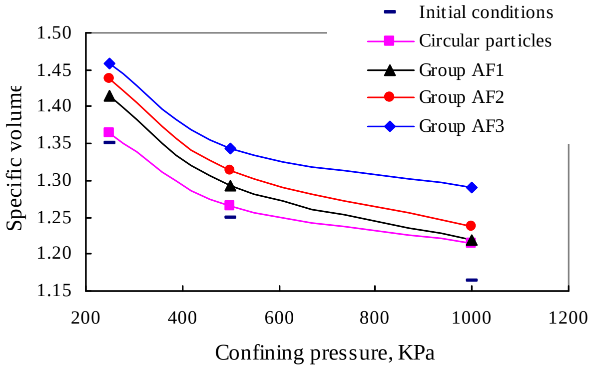

Figure 3. Critical state lines for different angularity groups

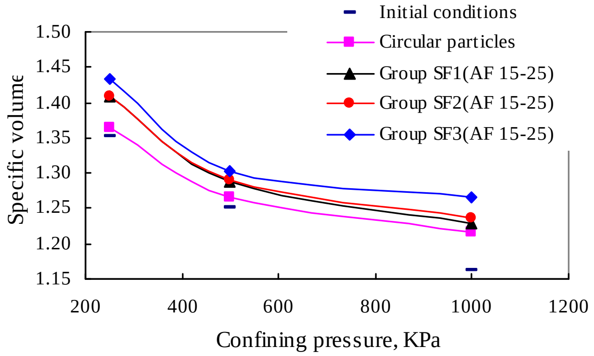

Figure 4. Critical state lines for different shape groups

4.2 *Effect of interparticle friction*

A numerical study of the effect of interparticle friction angle on the dilatancy and strength properties of angular soils was performed. Particles were modeled as discs and the interparticle friction coefficient varied from 0.25 to 2.50 under the same confining pressures. The shear strain versus both shear stress and volumetric strain were plotted for different interparticle coefficient. Figures 5 and 6 are examples of the resulted plots for friction coefficients of 0.25, 1.0, and 2.5. The results showed the peak strength increased with increasing interparticle friction, with the peak occurring at small strains. The rate of increase decreases for higher interparticle frictions. The critical state shear strength slightly increased with increasing interparticle friction and was almost identical for higher interparticle friction coefficients. The volumetric strain-shear strain plot showed the volumetric strain increased with the increase of the interparticle friction with decreasing rate for higher values. The maximum dilation angle was dependent on interparticle friction, but was almost identical for values higher than 1.5. The effect of interparticle friction on maximum volumetric strain, maximum dilation angle, and residual shearing resistance angle were established by fitting trend lines that showed R^2-values mostly above 95%.

4.3 *The equivalent interparticle friction coefficient*

The study showed that numerical simulations are very sensitive to the interparticle friction coefficient. The main obstacle to DEM is that simulations are time consuming especially with large number of actual shape particles. Unless the DEM is able to simulate millions of particles with their actual shapes, no real scale problems can be accurately simulated. Another alternative is to change some properties of circular particles to control the behavior to reproduce similar behavior to that of the angular particles since they are the most efficient shape to model.

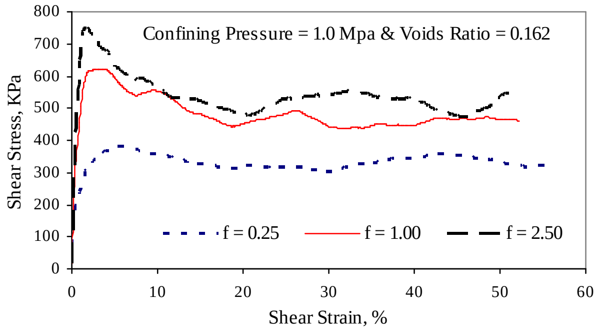

Figure 5. Shear stress-shear strain curves for different interparticle friction coefficients (circular particles)

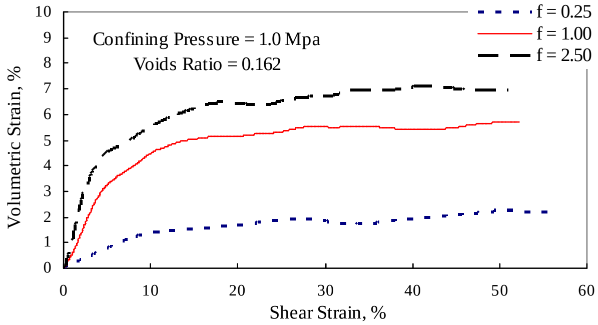

Figure 6. Volumetric strain-shear strain curves for different interparticle friction coefficients (circular particles)

Interparticle friction coefficient is one of the most significant properties in DEM simulations. The interparticle friction angle affects the position of the critical state line. It was observed also that the critical state lines for different angularity and shape groups were shifted up with respect to that of circular particles. To this extent, it may be possible to adjust the interparticle friction angle to account for the particle shape characteristics. The target interparticle friction coefficient can be extracted by re-plotting the critical state line for the required angularity or shape group on top of the critical state lines as shown on Figure 7 and choose the interparticle friction that matches the critical state line for the group.

The procedure was applied for the three angularity groups (AF1, AF2, and AF3) and the results showed that the critical state line for each angularity group varies as a function of friction values, so it would not be accurate to choose an equivalent value of friction to account for the angularity. One option is to continuously alter the friction coefficient as a function of v-$p^{/}$ state in order to match the response of the angular particles. The procedure was repeated for the three shape groups (SF1(AF 15-25), SF2(AF 15-25), and SF3(AF 15-25)) and the results were similar to Figure 7.

5 CONCLUSION

The effect of particle shape and angularity characteristics on strength and dilatancy properties was studied by performing discrete element numerical simulations of the triaxial test under pure shear conditions. A modification of the particle shape characterization procedure proposed by Sukumaran and Ashmawy (2001) was introduced. A total of 99 particles were divided into different particle shape and angularity groups. The modeled pure shear test was performed on circular particles as a reference and on different shape and angularity groups under three different initial conditions. Shear stress-shear strain and volumetric strain-shear strain curves were developed for circular and different particle group. The dependency of maximum volumetric strain, maximum dilation angle, and the resulted residual shear strength angle on shape and angularity factors was explored. Critical state lines for shape and angularity groups along with those of circular particle were evaluated. It was concluded that the critical state lines, for different shape and angularity groups, were shifted up compared with that of the circular particles, which indicates an increase of volumetric strain and hence dilation. The effect of interparticle friction coefficient on the behavior was studied. The critical state lines showed significant dependency on interparticle friction. An attempt was made to use an equivalent interparticle friction to model different particle shapes. It was concluded that there is no one-to-one equivalency between interparticle friction and shape or angularity. Instead, the interparticle friction must be continuously altered as a function of confining pressure and void ratio to achieve the required effect.

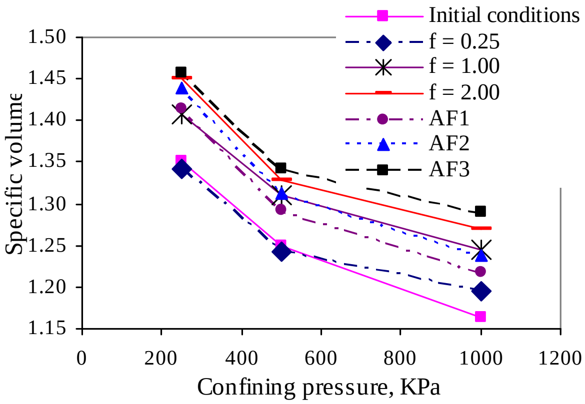

Figure 7. Critical state lines for circular particles (different interparticle friction coefficient) along with those for different angularity factor groups (f = 0.5)

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